Math Catch-Up Study Plan

Grade-Level Catch-Up Plan

Student age: 10
Plan duration: 1 week
Daily study time: 1 hour

Contents

1. How to Use This Plan

This plan is designed for one focused week of grade-level math practice to close key gaps. Here is how to get the most out of it.

1.1. Daily Structure

Each day follows a consistent rhythm:

Day	Focus
Day 1 – Day 4	One lesson introduction (theory + worked examples) followed by practice problems.
Day 5	Weekly review – no new concepts. Recap of all topics, mixed practice problems, and 2 challenge problems combining multiple topics.

1.2. How to Work Through a Lesson

1. **Read the theory section** fully before attempting any problems. Understand the concept, not just the formula.
2. **Study the worked examples** step by step. For each example, ask: "What was the key move here?" The **Key Insight** label names it explicitly.
3. **Attempt the practice problems yourself first** before reading the solution. Even a partial attempt builds understanding.
4. **Standard problems** build the core skill. Complete all of them.
5. **Challenge problems** (marked with a star ⋆) require an extra step or creative insight. Try them after the standard problems. It is fine to skip a challenge problem and return to it on Day 5.

1.3. When You Are Stuck

- Re-read the relevant theory section.
- Look at the most similar worked example and trace it step by step.
- If still stuck after 5 minutes on a standard problem, read the hint (if provided), then the solution.
- For challenge problems, being stuck is expected and normal. Write down what you **do** know, then look at the solution.

1.4. Problem-Solving Checklist

Use this checklist on any word problem or multi-step problem before you start calculating:

1. **What is the problem asking?** Identify the exact question.
2. **What do I know?** List the given numbers and facts.
3. **Is any information extra or distracting?** Not every number in a problem needs to be used.
4. **What should I find first?** Identify if there is a hidden intermediate value.
5. **What operation matches the *situation* (not just the keyword)?** "More" does not always mean add. Model the scenario.
6. **Does my answer make sense?** Check the magnitude and units of your result.

1.5. Progress Tracking

After each day, note which problems felt easy, which felt hard, and which felt impossible. This informs what topics to revisit and what to study next — the **What to Study Next** section gives you the natural continuation path.

2. Overview

2.1. Student Profile

2.1.1. Strengths

No specific strengths listed.

2.1.2. Focus Areas

- Long division

2.2. Why this plan?

This week's focus is on mastering long division, a critical skill for grade 4 math. The sequence progresses from understanding the division process to applying it in more complex problems, ensuring the student builds confidence and proficiency. By the end of the week, the student should be able to perform long division with greater accuracy and confidence.

2.3. Milestones

- The student will have a solid understanding of long division, including the ability to solve problems with single and multi-digit divisors, and apply divisibility rules effectively.

2.4. What to Study Next

After completing this plan, continue with the following topics:

- Explore word problems involving long division to apply skills in real-world contexts.
- Introduce decimals and how they relate to division.
- Reinforce multiplication skills to support division proficiency.

3. Schedule

Theme: Mastering Long Division

Topics: [long-division], [divisibility], [factors-and-multiples]

Goals:

- Understand and apply long division techniques
- Recognize divisibility rules
- Identify factors and multiples to aid division

Day	Activities
Day 1	Introduction to Long Division (30 min) · Practice Basic Long Division (30 min)
Day 2	Understanding Divisibility Rules (30 min) · Apply Divisibility in Division Problems (30 min)
Day 3	Factors and Multiples in Division (30 min) · Practice Long Division with Remainders (30 min)
Day 4	Complex Long Division Problems (30 min) · Practice Multi-Digit Long Division (30 min)
Day 5	Weekly Review (60 min)

4. Lessons

4.0.1. Day 1 — Long Division as Organized Sharing

You are starting Week 1 by rebuilding long division from the ground up, and that is a great place to begin. In Grade 4, long division helps you share large numbers fairly, make equal groups, and solve word problems without guessing. We will go slowly, use the same steps every time, and check each answer so the process starts to feel dependable.

4.0.1.1. Theory

Long division is organized sharing or organized grouping. When you see $84 \div 4$, the number being divided is the dividend, so 84 is the dividend. The number you divide by is the divisor, so 4 is the divisor. The answer is the quotient. If something is left over, that amount is the remainder.

Division and multiplication are inverse operations. That means they undo each other. If $6 \times 7 = 42$, then $42 \div 6 = 7$ and $42 \div 7 = 6$. A helpful question during division is: What times the divisor gets close to this part without going over?

Use the Five D's for every long division problem:

1. Decode: Name the dividend, divisor, and what the problem is asking.
2. Estimate: Make a quick guess so you know about how large the answer should be.
3. Divide: Use the DMSB loop: Divide, Multiply, Subtract, Bring down. Repeat until there are no digits left.
4. Decide: In word problems, decide what the remainder means.
5. Double-check: Use multiplication to check your answer.

The DMSB loop is the heart of long division: Divide: How many times does the divisor fit in the current part? Multiply: Multiply the divisor by the quotient digit you chose. Subtract: Find what is left. Bring down: Bring down the next digit. Then repeat.

Worked inside example: $68 \div 4$. Estimate first: 68 is close to 80, and $80 \div 4 = 20$, so the answer should be near 20 but a little less. Now divide from left to right. 6 tens $\div 4 = 1$ ten. Write 1 in the tens place. Multiply: $1 \times 4 = 4$. Subtract: $6 - 4 = 2$. Bring down the 8 to make 28. Now $28 \div 4 = 7$. Write 7 in the ones place. The answer is 17. Check: $17 \times 4 = 68$, so $68 \div 4 = 17$.

Sometimes the divisor does not divide evenly. For example, $29 \div 6 = 4$ remainder 5, because $6 \times 4 = 24$ and $29 - 24 = 5$. The remainder must always be smaller than the divisor. For division by 6, the remainder can be 0, 1, 2, 3, 4, or 5, but not 6.

In word problems, do not just look for a keyword and guess an operation. Picture the situation. Are you sharing into equal groups, making groups of a certain size, finding leftovers, or making sure there is enough for everyone? The same division, $29 \div 6 = 4$ remainder 5, can mean 4 full bags with 5 left over, or 5 leftovers, or 5 tables needed if everyone must have a seat. The story tells you what to do with the remainder.

To check long division, use multiplication. If there is no remainder, divisor $\times$ quotient = dividend. If there is a remainder, divisor $\times$ quotient + remainder = dividend.

4.0.1.2. Worked Examples

4.0.1.3. Example 1

Problem: Find $84 \div 4$.

Solution:

Step 1: Decode. The dividend is 84 and the divisor is 4. Step 2: Estimate. $80 \div 4 = 20$, so the answer should be near 20. Step 3: Divide the tens. $8 \div 4 = 2$. Write 2 in the tens place. Step 4: Multiply. $2 \times 4 = 8$. Step 5: Subtract. $8 - 8 = 0$. Step 6: Bring down the 4. Step 7: Divide the ones. $4 \div 4 = 1$. Write 1 in the ones place. Step 8: Check. $21 \times 4 = 84$. Answer: 21.

Key insight: *Work from left to right and use Divide, Multiply, Subtract, Bring down.*

4.0.1.4. Example 2

Problem: Find $156 \div 6$.

Solution:

Step 1: Decode. The dividend is 156 and the divisor is 6. Step 2: Estimate. 156 is close to 150, and $150 \div 6 = 25$, so the answer should be near 25. Step 3: Start at the left. 1 hundred is not enough to divide by 6, so use 15 tens. Step 4: Divide. $15 \div 6 = 2$ with some left over. Write 2. Step 5: Multiply. $2 \times 6 = 12$. Step 6: Subtract. $15 - 12 = 3$. Step 7: Bring down the 6 to make 36. Step 8: Divide. $36 \div 6 = 6$. Write 6. Step 9: Check. $26 \times 6 = 156$. Answer: 26.

Key insight: *If the first digit is too small, use the first two digits and keep the quotient lined up by place value.*

4.0.1.5. Example 3

Problem: Find $824 \div 4$.

Solution:

Step 1: Decode. The dividend is 824 and the divisor is 4. Step 2: Estimate. $800 \div 4 = 200$, so the answer should be near 200. Step 3: Divide. $8 \div 4 = 2$. Write 2 in the hundreds place. Step 4: Multiply. $2 \times 4 = 8$. Step 5: Subtract. $8 - 8 = 0$. Step 6: Bring down the 2. Step 7: Divide. $2 \div 4 = 0$. Write 0 in the tens place because there are no groups of 4 in 2 tens. Step 8: Bring down the 4 to make 24. Step 9: Divide. $24 \div 4 = 6$. Write 6 in the ones place. Step 10: Check. $206 \times 4 = 824$. Answer: 206.

Key insight: *A place with no groups still needs a 0 in the quotient.*

4.0.1.6. Practice Problems

1. **Find $48 \div 4$.**

   Solution: Step 1: Estimate. $40 \div 4 = 10$, so the answer should be a little more than 10. Step 2: Divide the tens. $4 \div 4 = 1$. Write 1 in the tens place. Step 3: Multiply and subtract. $1 \times 4 = 4$, and $4 - 4 = 0$. Step 4: Bring down 8. Step 5: Divide the ones. $8 \div 4 = 2$. Write 2. Step 6: Check. $12 \times 4 = 48$. Answer: 12.

2. **Find $91 \div 7$.**

   Solution: Step 1: Estimate. $70 \div 7 = 10$, so the answer should be a little more than 10. Step 2: Divide. $9 \div 7 = 1$. Write 1 in the tens place. Step 3: Multiply. $1 \times 7 = 7$. Step 4:

Subtract. $9 - 7 = 2$. Step 5: Bring down the 1 to make 21. Step 6: Divide. $21 \div 7 = 3$. Write 3 in the ones place. Step 7: Check. $13 \times 7 = 91$. Answer: 13.

3. **A teacher has 72 markers and puts them equally into 6 bins. How many markers are in each bin? Then check your answer with multiplication.**

   Solution: Step 1: Decode. This is equal sharing: 72 markers shared into 6 bins. Step 2: Divide. $72 \div 6 = 12$. Step 3: Check with multiplication. $12 \times 6 = 72$. Answer: 12 markers in each bin.

4. **Find $612 \div 3$.** *Hint: Watch the tens place. If 1 ten cannot make a group of 3, write 0 in the tens place before bringing down the next digit.*

   Solution: Step 1: Estimate. $600 \div 3 = 200$, so the answer should be near 200. Step 2: Divide. $6 \div 3 = 2$. Write 2 in the hundreds place. Step 3: Multiply and subtract. $2 \times 3 = 6$, and $6 - 6 = 0$. Step 4: Bring down the 1. Step 5: Divide. $1 \div 3 = 0$. Write 0 in the tens place. Step 6: Bring down the 2 to make 12. Step 7: Divide. $12 \div 3 = 4$. Write 4 in the ones place. Step 8: Check. $204 \times 3 = 612$. Answer: 204.

5. **125 students ride in vans that hold 6 students each. How many vans are needed so every student has a seat? Explain your solution in 2–3 sentences. Use this format: I know _____. I need to find _____. First I _____ because _____. My answer makes sense because _____.** *Hint: First find $125 \div 6$. Then decide what the remainder means when every student needs a seat.*

   Solution: Step 1: Decode. This is grouping: groups of 6 students per van. Step 2: Divide. $125 \div 6 = 20$ remainder 5, because $6 \times 20 = 120$ and $125 - 120 = 5$. Step 3: Decide what the remainder means. There are 5 students left after filling 20 vans, and they still need seats. Step 4: Add 1 more van. $20 + 1 = 21$. Step 5: Check reasonableness. 20 vans hold 120 students, which is not enough. 21 vans hold 126 students, which is enough. Answer: 21 vans.

4.0.1.7. Study Tips

- For practice, say the DMSB steps out loud: Divide, Multiply, Subtract, Bring down. This helps prevent skipped steps.
- After every long division problem, check with multiplication. If there is a remainder, use divisor $\times$ quotient + remainder = dividend.
- You will know this skill is getting stronger when your estimate and exact answer are close, your remainder is smaller than the divisor, and you can explain what the answer means in a word problem.

4.0.2. Day 2 — Divisibility Means No Remainder

You are rebuilding an important foundation for long division: knowing when one number divides evenly into another. This skill helps you catch mistakes early because you can tell whether a division answer should have a remainder or not. We will go slowly and use multiplication facts to make divisibility feel clear instead of mysterious.

4.0.2.1. Theory

Divisibility means that one number can be divided by another number with no remainder. For example, 48 is divisible by 6 because $48 \div 6 = 8$ exactly. Another way to say the same thing is $6 \times 8 = 48$, so multiplication can check division. The number being divided is the dividend. The number you divide by is the divisor. The answer is the quotient. If there is something left over, that leftover amount is the remainder. A number is divisible by the divisor only when the remainder is 0. Key rule: the remainder must always be smaller than the divisor. For example, when dividing by 5, the only possible remainders are 0, 1, 2, 3, or 4. A remainder of 5 would mean you could make one more group of 5. To test divisibility, ask: "Can I multiply the divisor by a whole number to get the dividend exactly?" If yes, it is divisible. If no, divide and find the remainder. Worked example inside the lesson: Is 72 divisible by 8? Think of the multiplication fact: $8 \times 9 = 72$. So $72 \div 8 = 9$ with no remainder. Since the remainder is 0, 72 is divisible by 8. If you tested $74 \div 8$, $8 \times 9 = 72$ and $74 - 72 = 2$, so $74 \div 8 = 9$ remainder 2. Since there is a remainder, 74 is not divisible by 8. For long division, you can use the Five D's: Decode the dividend and divisor, Estimate the size of the answer, Divide using DMSB, Decide what the remainder means in a story, and Double-check with multiplication.

4.0.2.2. Worked Examples

4.0.2.3. Example 1

Problem: Is 36 divisible by 4? If it is, what is the quotient?

Solution:

Step 1: The dividend is 36 and the divisor is 4. Step 2: Use a multiplication fact to check: $4 \times 9 = 36$. Step 3: Since $36 \div 4 = 9$ with no remainder, the remainder is 0. Step 4: Yes, 36 is divisible by 4. The quotient is 9.

Key insight: *A number is divisible by another number when the division has remainder 0.*

4.0.2.4. Example 2

Problem: Is 85 divisible by 6? If not, write the quotient and remainder.

Solution:

Step 1: The dividend is 85 and the divisor is 6. Step 2: Estimate: $85 \div 6$ is close to $84 \div 6 = 14$, so the quotient should be about 14. Step 3: Check $6 \times 14 = 84$. Step 4: Subtract to find what is left: $85 - 84 = 1$. Step 5: So $85 \div 6 = 14$ remainder 1. Step 6: The remainder 1 is smaller than the divisor 6, so the remainder makes sense. Since the remainder is not 0, 85 is not divisible by 6.

Key insight: *If multiplication gets close but not exact, the leftover amount is the remainder.*

4.0.2.5. Practice Problems

1. **Is 42 divisible by 6? If it is, what is the quotient?**

   Solution: Step 1: Check the multiplication fact: $6 \times 7 = 42$. Step 2: Since $42 \div 6 = 7$ with no remainder, the remainder is 0. Step 3: Yes, 42 is divisible by 6. The quotient is 7.

2. **Is 57 divisible by 4? If not, write the quotient and remainder.**

   Solution: Step 1: Find the closest multiple of 4 that is less than or equal to 57. Step 2: $4 \times 14 = 56$. Step 3: Subtract: $57 - 56 = 1$. Step 4: So $57 \div 4 = 14$ remainder 1. Step 5: Since the remainder is not 0, 57 is not divisible by 4.

3. **There are 64 crayons. A teacher puts 8 crayons in each box. Is 64 divisible by 8, and how many full boxes can the teacher make? Explain your solution in 2–3 sentences. Use this format: I know _____. I need to find _____. First I _____ because _____. My answer makes sense because _____.**

   Solution: Step 1: The total number of crayons is 64, and each box has 8 crayons. Step 2: Check $64 \div 8$ using multiplication: $8 \times 8 = 64$. Step 3: Since the remainder is 0, 64 is divisible by 8. Step 4: The teacher can make 8 full boxes. Example explanation: I know there are 64 crayons and 8 crayons go in each box. I need to find whether 64 divides evenly by 8 and how many boxes that makes. First I checked $8 \times 8 = 64$ because multiplication checks division. My answer makes sense because there are no crayons left over.

4. **A teacher has 95 stickers and wants to make equal groups of 7 stickers. Is 95 divisible by 7? If not, how many full groups can be made and how many stickers are left over?** *Hint: Find the largest multiple of 7 that is less than or equal to 95.*

   Solution: Step 1: Estimate: $95 \div 7$ is close to $98 \div 7 = 14$, so try 13 or 14. Step 2: $7 \times 13 = 91$, and $7 \times 14 = 98$, which is too high. Step 3: Use 91 and subtract: $95 - 91 = 4$. Step 4: So $95 \div 7 = 13$ remainder 4. Step 5: Since the remainder is not 0, 95 is not divisible by 7. The teacher can make 13 full groups with 4 stickers left over.

5. **A school has 126 notebooks to put into stacks of 6 notebooks. Is 126 divisible by 6? If it is, how many stacks can be made? Show a multiplication check.** *Hint: Use long division or think: $6 \times 20 = 120$, then see what is left.*

   Solution: Step 1: Estimate: $126 \div 6$ is close to $120 \div 6 = 20$, so the answer should be a little more than 20. Step 2: Use the hint: $6 \times 20 = 120$. Step 3: Find what is left: $126 - 120 = 6$. Step 4: One more group of 6 makes $6 \times 21 = 126$. Step 5: So $126 \div 6 = 21$ with no remainder. Step 6: Since the remainder is 0, 126 is divisible by 6. The school can make 21 stacks. Multiplication check: $6 \times 21 = 126$.

4.0.2.6. Study Tips

- Before deciding if a number is divisible, always ask: "Can the divisor multiply by a whole number to make the dividend exactly?" This connects division to multiplication and makes the work feel less like guessing.
- Check every remainder. If the remainder is 0, the number is divisible. If the remainder is bigger than or equal to the divisor, go back because one more group can still be made.

4.0.3. Day 3 — Using Factors and Multiples to Make Long Division Easier

You are working on long division, and today we are going to rebuild one of the most important foundations: factors and multiples. When you know how multiplication facts connect to division, long division starts to feel less like a mystery and more like organized sharing. This lesson will help you choose division steps more confidently and check that your answers make sense.

4.0.3.1. Theory

Division and multiplication are opposites. If $6 \times 7 = 42$, then $42 \div 6 = 7$ and $42 \div 7 = 6$. These three facts belong to the same fact family.

A factor is a number that is multiplied by another number to make a product. In $6 \times 7 = 42$, the numbers 6 and 7 are factors of 42. A multiple is made by multiplying. The multiples of 5 are 5, 10, 15, 20, 25, and they keep going forever.

Factors and multiples help with division because division asks, What times this equals that? For example, to solve $48 \div 6$, ask: $6 \times ? = 48$. Count multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48. Since $6 \times 8 = 48$, we know $48 \div 6 = 8$. We can check by multiplying: $8 \times 6 = 48$.

In long division, you often do not use the whole number at once. You look at part of the number and ask for the largest multiple that fits. For example, in $156 \div 4$, first look at 15. The multiples of 4 are 4, 8, 12, 16. The largest multiple of 4 that is not over 15 is 12, which is 4×3. That means the first quotient digit is 3. Subtract 12 from 15 to get 3, then bring down the 6 to make 36. Since $4 \times 9 = 36$, the next quotient digit is 9. So $156 \div 4 = 39$. Check: $39 \times 4 = 156$.

For word problems, do not only look for key words. Instead, picture the situation: What is the total? What is the size of each group? What do I need to find? If you know the total and the size of each equal group, division can tell you how many groups there are.

4.0.3.2. Worked Examples

4.0.3.3. Example 1

Problem: Use multiples to solve $54 \div 6$.

Solution:

Step 1: Ask, $6 \times ? = 54$. Step 2: List multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54. Step 3: 54 is the 9th multiple of 6, so $6 \times 9 = 54$. Step 4: Therefore, $54 \div 6 = 9$. Step 5: Check: $9 \times 6 = 54$.

Key insight: *Division facts come from multiplication facts: ask, What times the divisor equals the dividend?*

4.0.3.4. Example 2

Problem: Use multiples of 4 to divide: $156 \div 4$.

Solution:

Step 1: Look at 15 first because 4 does not go into 1. Step 2: Find the largest multiple of 4 that is not over 15: 4, 8, 12, 16. Use 12 because 16 is too large. Step 3: $4 \times 3 = 12$, so write 3 in the tens place. Step 4: Subtract: 15 - 12 = 3. Step 5: Bring down the 6 to make 36. Step 6: 4×9

= 36, so write 9 in the ones place. Step 7: Subtract: 36 - 36 = 0. Step 8: The quotient is 39. Check: 39 × 4 = 156.

Key insight: *In long division, use the largest multiple of the divisor that fits without going over.*

4.0.3.5. Practice Problems

1. **List the first five multiples of 7. Then use the list to solve 35 ÷ 7.**

   Solution: Step 1: The first five multiples of 7 are 7, 14, 21, 28, 35. Step 2: 35 is the 5th multiple of 7. Step 3: So 7 × 5 = 35. Step 4: Therefore, 35 ÷ 7 = 5.

2. **Is 6 a factor of 48? Use a multiplication fact to explain your answer.**

   Solution: Step 1: Ask, 6 × ? = 48. Step 2: 6 × 8 = 48. Step 3: Since 6 can be multiplied by 8 to make 48, 6 is a factor of 48.

3. **A teacher has 56 pencils. She puts 8 pencils in each cup. How many cups can she fill?**

   Solution: Step 1: The total is 56 pencils. Step 2: Each group has 8 pencils. Step 3: Divide to find the number of groups: 56 ÷ 8. Step 4: Use the fact 8 × 7 = 56. Step 5: So 56 ÷ 8 = 7. She can fill 7 cups.

4. **Use long division and multiples of 6 to solve 168 ÷ 6. Explain your solution in 2–3 sentences. Use this format: I know _____. I need to find _____. First I _____ because _____. My answer makes sense because _____.** *Hint: Start with 16. The largest multiple of 6 that is not over 16 is 12.*

   Solution: Step 1: Look at 16. The largest multiple of 6 that is not over 16 is 12. Step 2: 6 × 2 = 12, so write 2 in the tens place. Step 3: Subtract: 16 - 12 = 4. Step 4: Bring down the 8 to make 48. Step 5: 6 × 8 = 48, so write 8 in the ones place. Step 6: Subtract: 48 - 48 = 0. Step 7: The quotient is 28. Check: 28 × 6 = 168. Example explanation: I know 168 is being divided into groups of 6. I need to find how many groups of 6 are in 168. First I used the largest multiple of 6 that fit into 16 because long division works one place at a time. My answer makes sense because 28 × 6 = 168.

5. **A school has 208 markers. Each box holds 4 markers. How many boxes are needed to hold all the markers?** *Hint: Model the situation: 208 is the total, 4 is the size of each group, and you need the number of groups. Divide 208 ÷ 4.*

   Solution: Step 1: The total is 208 markers. Step 2: Each box holds 4 markers, so divide: 208 ÷ 4. Step 3: Look at 20. Since 4 × 5 = 20, write 5 in the tens place. Step 4: Subtract: 20 - 20 = 0. Step 5: Bring down the 8. Step 6: Since 4 × 2 = 8, write 2 in the ones place. Step 7: Subtract: 8 - 8 = 0. Step 8: 208 ÷ 4 = 52. The school needs 52 boxes. Check: 52 × 4 = 208.

4.0.3.6. Study Tips

- When you get stuck on a division fact, turn it into a multiplication question: What times this number equals the total?
- After every division problem, check by multiplying the quotient by the divisor. If you get the original number, your answer is on track.

4.0.4. Day 4 — Complex Long Division: Zeros, Remainders, and Checking

You are working on catching up in long division, and today we will rebuild the steps slowly and clearly. In Grade 4, long division helps you solve bigger sharing and grouping problems without guessing. Complex problems may have zeros in the quotient or remainders, but they still follow the same dependable steps every time.

4.0.4.1. Theory

Long division is organized sharing. A division problem has a dividend, a divisor, a quotient, and sometimes a remainder. In $57 \div 4 = 14$ remainder 1, 57 is the dividend, 4 is the divisor, 14 is the quotient, and 1 is the remainder. The remainder must always be smaller than the divisor.

Use the Five D's every time:

1. Decode: Read the problem. Name the dividend and divisor. Decide what the question is asking: sharing, grouping, full groups, leftovers, or how many are needed.
2. Estimate: Find a rough answer before doing long division. This helps you notice if your final answer is impossible.
3. Divide: Use DMSB: Divide, Multiply, Subtract, Bring down. Repeat until no digits are left.
4. Decide: In word problems, decide what the remainder means. Sometimes you use the quotient, sometimes the remainder, and sometimes you round up.
5. Double-check: Check with multiplication. If there is a remainder, use divisor $\times$ quotient $+$ remainder $=$ dividend.

Example from the foundations: Solve $735 \div 6$. Estimate first: 735 is close to 720, and $720 \div 6 = 120$, so the answer should be near 120. Now divide: 6 goes into 7 one time. Write 1. Multiply: $1 \times 6 = 6$. Subtract: $7 - 6 = 1$. Bring down 3 to make 13. 6 goes into 13 two times. Write 2. Multiply: $2 \times 6 = 12$. Subtract: $13 - 12 = 1$. Bring down 5 to make 15. 6 goes into 15 two times. Write 2. Multiply: $2 \times 6 = 12$. Subtract: $15 - 12 = 3$. No more digits remain, so $735 \div 6 = 122$ remainder 3. Check: $6 \times 122 + 3 = 732 + 3 = 735$. The remainder 3 is smaller than 6, so it works.

A very important long-division idea is the zero in the quotient. If a place has no groups, you must write 0 in that place. For example, in $824 \div 4$, after $8 \div 4 = 2$, you bring down 2. Since $2 \div 4 = 0$, you write 0 in the tens place. Then bring down 4 to make 24. The answer is 206, not 26.

For word problems, do not just look for one keyword. Picture the situation. If 74 students need vans that hold 8 students each, $74 \div 8 = 9$ remainder 2. You cannot leave 2 students without a van, so you need 10 vans. The story decides what to do with the remainder.

4.0.4.2. Worked Examples

4.0.4.3. Example 1

Problem: Solve $824 \div 4$.

Solution:

1. Estimate: 824 is close to 800, and $800 \div 4 = 200$, so the answer should be near 200.
2. Divide: $8 \div 4 = 2$. Write 2 in the hundreds place.
3. Multiply: $2 \times 4 = 8$.
4. Subtract: $8 - 8 = 0$.

5. Bring down 2. Now solve $2 \div 4$. Since 4 does not fit into 2, write 0 in the tens place.
6. Multiply: $0 \times 4 = 0$.
7. Subtract: $2 - 0 = 2$.
8. Bring down 4 to make 24.
9. Divide: $24 \div 4 = 6$. Write 6 in the ones place.
10. Check: $206 \times 4 = 824$.

Answer: 206.

Key insight: *A place with no groups still needs a 0 in the quotient.*

4.0.4.4. Example 2

Problem: A school has 157 notebooks. Each box holds 6 notebooks. How many boxes are needed to hold all the notebooks?

Solution:

1. Decode: The dividend is 157 notebooks. The divisor is 6 notebooks per box. We need boxes for all notebooks.
2. Estimate: 157 is close to 150, and $150 \div 6 = 25$, so the answer should be a little more than 25 boxes.
3. Divide: $15 \div 6 = 2$. Write 2.
4. Multiply: $2 \times 6 = 12$.
5. Subtract: $15 - 12 = 3$.
6. Bring down 7 to make 37.
7. Divide: $37 \div 6 = 6$. Write 6.
8. Multiply: $6 \times 6 = 36$.
9. Subtract: $37 - 36 = 1$.
10. So $157 \div 6 = 26$ remainder 1.
11. Decide: 26 full boxes hold 156 notebooks, but 1 notebook is left. To hold all the notebooks, one more box is needed.
12. Check: $6 \times 26 + 1 = 156 + 1 = 157$.

Answer: 27 boxes.

Key insight: *When the question asks how many are needed for everything, a remainder means you round up.*

4.0.4.5. Practice Problems

1. **Solve $612 \div 3$.**

   Solution: 1. Estimate: $600 \div 3 = 200$, so the answer should be near 200.
   2. $6 \div 3 = 2$. Write 2.
   3. $2 \times 3 = 6$. Subtract: $6 - 6 = 0$.
   4. Bring down 1. Since $1 \div 3 = 0$, write 0 in the tens place.
   5. $0 \times 3 = 0$. Subtract: $1 - 0 = 1$.
   6. Bring down 2 to make 12.
   7. $12 \div 3 = 4$. Write 4.
   8. Check: $204 \times 3 = 612$.

   Answer: 204.

2. **Solve $935 \div 4$.**

Solution: 1. Estimate: 935 is close to 920, and $920 \div 4 = 230$, so the answer should be near 230.

2. $9 \div 4 = 2$. Write 2.
3. $2 \times 4 = 8$. Subtract: $9 - 8 = 1$.
4. Bring down 3 to make 13.
5. $13 \div 4 = 3$. Write 3.
6. $3 \times 4 = 12$. Subtract: $13 - 12 = 1$.
7. Bring down 5 to make 15.
8. $15 \div 4 = 3$. Write 3.
9. $3 \times 4 = 12$. Subtract: $15 - 12 = 3$.
10. The remainder is 3, and 3 is smaller than 4.
11. Check: $4 \times 233 + 3 = 932 + 3 = 935$.

Answer: 233 remainder 3.

3. **There are 186 stickers shared equally among 6 students. Then each student gives away 5 stickers. How many stickers does each student have left?**

Solution: 1. First find how many stickers each student gets: $186 \div 6$.

2. Estimate: $180 \div 6 = 30$, so the answer should be near 30.
3. $18 \div 6 = 3$. Write 3.
4. $3 \times 6 = 18$. Subtract: $18 - 18 = 0$.
5. Bring down 6.
6. $6 \div 6 = 1$. Write 1.
7. So $186 \div 6 = 31$.
8. Each student gives away 5, so subtract: $31 - 5 = 26$.

Answer: 26 stickers.

4. **There are 74 students going on a trip. Each table in the lunch room seats 6 students. How many tables are needed? Explain your solution in 2–3 sentences. Use this format: I know _____. I need to find _____. First I _____ because _____. My answer makes sense because _____.** *Hint: After you divide, ask yourself whether the leftover students still need a place to sit.*

Solution: 1. Decode: The dividend is 74 students. The divisor is 6 students per table. We need enough tables for every student.

2. Estimate: $72 \div 6 = 12$, so the answer should be a little more than 12 if there are leftovers.
3. $7 \div 6 = 1$. Write 1.
4. $1 \times 6 = 6$. Subtract: $7 - 6 = 1$.
5. Bring down 4 to make 14.
6. $14 \div 6 = 2$. Write 2.
7. $2 \times 6 = 12$. Subtract: $14 - 12 = 2$.
8. So $74 \div 6 = 12$ remainder 2.
9. Decide: 12 tables seat 72 students, but 2 students are left. They need another table.

Answer: 13 tables. Sample explanation: I know there are 74 students and each table seats 6 students. I need to find how many tables are needed for everyone. First I divided 74 by 6 because the students are being put into equal groups of 6. My answer makes sense because 12 tables leave 2 students without seats, so 13 tables are needed.

5. **A library has 458 books to pack into boxes. Each box holds 8 books. If the library fills as many full boxes as possible, how many full boxes will there be, how many books will be left, and how many more books would be needed to fill one more box?** *Hint: Use the quotient for full boxes and the remainder for books left. Then compare the remainder to 8.*

   Solution: 1. Decode: The dividend is 458 books. The divisor is 8 books per box.
   2. Estimate: $480 \div 8 = 60$, so the answer should be a little less than 60 full boxes.
   3. $45 \div 8 = 5$. Write 5.
   4. $5 \times 8 = 40$. Subtract: $45 - 40 = 5$.
   5. Bring down 8 to make 58.
   6. $58 \div 8 = 7$. Write 7.
   7. $7 \times 8 = 56$. Subtract: $58 - 56 = 2$.
   8. So $458 \div 8 = 57$ remainder 2.
   9. Decide: There are 57 full boxes and 2 books left.
   10. One full box needs 8 books. Since there are already 2 books left, $8 - 2 = 6$ more books are needed to fill one more box.
   11. Check: $8 \times 57 + 2 = 456 + 2 = 458$.

   Answer: 57 full boxes, 2 books left, and 6 more books needed.

4.0.4.6. Study Tips

- Say DMSB quietly as you work: Divide, Multiply, Subtract, Bring down. This helps prevent skipped steps, especially on longer problems.
- Always do two checks: make sure the remainder is smaller than the divisor, and check with divisor $\times$ quotient + remainder = dividend.

4.0.5. Day 5 — Weekly Review: Long Division, Divisibility, Factors, and Multiples

This week, you rebuilt the foundations of long division: sharing in equal groups, using multiplication to check division, and making sense of remainders. You also practiced divisibility, factors, and multiples so division feels less like guessing and more like organized thinking. Today is a mixed review to help you see what you can now do with more confidence.

4.0.5.1. Theory

Quick review sheet:

- Division and multiplication are inverse operations. If $7 \times 8 = 56$, then $56 \div 7 = 8$ and $56 \div 8 = 7$.
- In long division, use DMSB: Divide, Multiply, Subtract, Bring down. Repeat until there are no more digits to bring down.
- The remainder rule: the remainder must be less than the divisor. If the remainder is equal to or greater than the divisor, another group still fits.
- Check long division with: divisor $\times$ quotient + remainder = dividend. For example, $5 \times 14 + 2 = 72$, so $72 \div 5 = 14$ r 2.
- A zero placeholder matters. If a place has no groups, write 0 in the quotient so the place value stays correct.
- Divisibility rules help you decide quickly if a number divides evenly. A number is divisible by 2 if it is even, by 5 if it ends in 0 or 5, and by 10 if it ends in 0.
- Factors multiply to make a number. For 12, factors include 1, 2, 3, 4, 6, and 12.
- Multiples are what you get by skip-counting by a number. Multiples of 6 include 6, 12, 18, 24, and 30.
- Prime numbers have exactly two factors: 1 and themselves. Composite numbers have more than two factors.
- In word problems, do not just look for keywords. Picture what is happening: Are items being shared equally? Are you making equal groups? Does the leftover matter?

4.0.5.2. Worked Examples

4.0.5.3. Example 1

Problem: Find all the factors of 24. Then decide whether 24 is prime or composite.

Solution:

1. Start with factor pairs that multiply to 24.
2. $1 \times 24 = 24$, so 1 and 24 are factors.
3. $2 \times 12 = 24$, so 2 and 12 are factors.
4. $3 \times 8 = 24$, so 3 and 8 are factors.
5. $4 \times 6 = 24$, so 4 and 6 are factors.
6. List them in order: 1, 2, 3, 4, 6, 8, 12, 24.
7. Since 24 has more than two factors, 24 is composite.

Key insight: *Use factor pairs systematically so you do not skip any factors.*

4.0.5.4. Example 2

Problem: Solve $408 \div 4$ using long division. Then check your answer.

Solution:

1. Divide the hundreds: $4 \div 4 = 1$, so write 1 in the hundreds place.
2. Multiply: $1 \times 4 = 4$.
3. Subtract: $4 - 4 = 0$.
4. Bring down the 0 tens.
5. $0 \div 4 = 0$, so write 0 in the tens place. This zero is important.
6. Multiply: $0 \times 4 = 0$.
7. Subtract: $0 - 0 = 0$.
8. Bring down the 8 ones.
9. $8 \div 4 = 2$, so write 2 in the ones place.
10. The quotient is 102.
11. Check: $4 \times 102 = 408$, so the answer is correct.

Key insight: *When a place has no groups, write 0 in the quotient as a placeholder.*

4.0.5.5. Practice Problems

1. **List all the factors of 16. Then decide whether 16 is prime or composite.**

   Solution: 1. Find factor pairs for 16.
   2. $1 \times 16 = 16$, so 1 and 16 are factors.
   3. $2 \times 8 = 16$, so 2 and 8 are factors.
   4. $4 \times 4 = 16$, so 4 is a factor.
   5. The factors are 1, 2, 4, 8, 16.
   6. Since 16 has more than two factors, 16 is composite.

2. **For each number, say whether it is divisible by 2, 5, and 10: 84, 95, 120.**

   Solution: 1. 84 ends in 4, so it is divisible by 2. It does not end in 0 or 5, so it is not divisible by 5 or 10.
   2. 95 ends in 5, so it is divisible by 5. It is not even, so it is not divisible by 2. It does not end in 0, so it is not divisible by 10.
   3. 120 ends in 0, so it is divisible by 10. Any number ending in 0 is also divisible by 2 and 5.
   4. Answers: 84: divisible by 2 only. 95: divisible by 5 only. 120: divisible by 2, 5, and 10.

3. **Solve $156 \div 3$ using long division. Explain your solution in 2–3 sentences. Use this format: I know _____. I need to find _____. First I _____ because _____. My answer makes sense because _____.**

   Solution: 1. Divide the hundreds: $1 \div 3$ does not make a whole group, so use 15 tens.
   2. $15 \div 3 = 5$, so write 5 in the tens place.
   3. Multiply: $5 \times 3 = 15$.
   4. Subtract: $15 - 15 = 0$.
   5. Bring down the 6.
   6. $6 \div 3 = 2$, so write 2 in the ones place.
   7. The quotient is 52.
   8. Check: $3 \times 52 = 156$, so $156 \div 3 = 52$.
   9. Sample explanation: I know 156 is being divided into 3 equal groups. I need to find how many are in each group. First I divided 15 tens by 3 because 1 hundred alone was not enough to make 3 hundreds. My answer makes sense because $3 \times 52 = 156$.

4. **A teacher has 125 pencils and packs them into boxes of 6 pencils each. How many full boxes can the teacher make, and how many pencils are left over?** *Hint: Model this as making equal groups of 6. The remainder tells how many pencils are left after all full boxes are made.*

   Solution: 1. Divide $125 \div 6$.
   2. 6 goes into 12 two times because $2 \times 6 = 12$.
   3. Subtract: $12 - 12 = 0$.
   4. Bring down the 5.
   5. 6 goes into 5 zero times, so the quotient is 20 with 5 left over.
   6. Check: $6 \times 20 + 5 = 120 + 5 = 125$.
   7. The teacher can make 20 full boxes, with 5 pencils left over.

5. **There are 248 chairs. First, the chairs are placed equally into 4 rows. Later, all 248 chairs are stored in stacks of 10. How many chairs are in each row? How many full stacks of 10 can be made, and how many chairs are left over?** *Hint: This has two separate division questions: 248 ÷ 4 for the rows, and 248 ÷ 10 for the stacks.*

   Solution: 1. First find the number of chairs in each row: $248 \div 4$.
   2. 4 goes into 24 six times because $6 \times 4 = 24$.
   3. Subtract: $24 - 24 = 0$.
   4. Bring down the 8.
   5. $8 \div 4 = 2$.
   6. So $248 \div 4 = 62$. There are 62 chairs in each row.
   7. Now find the number of stacks of 10: $248 \div 10$.
   8. $10 \times 24 = 240$, and $248 - 240 = 8$.
   9. So $248 \div 10 = 24$ r 8.
   10. There can be 24 full stacks of 10, with 8 chairs left over.

4.0.5.6. Study Tips

- When you finish a division problem, check it with divisor × quotient + remainder = dividend. This catches most long-division mistakes.
- For factors, list factor pairs in order: start with 1 × the number, then 2, then 3, and keep going until the pairs meet or pass each other.